A New Methodology for Robust Optimizations of Inverse Problems under Interval Uncertainty

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To consider the interval uncertainty in a practical inverse problem, a new methodology for efficient robust optimizations is proposed. The proposed methodology uses a constrained formulation for robust performances not only in alleviating the inefficiency of existing approaches in modeling interval uncertainties but also in avoiding the deficiency in the biasing force selection. The gradient information is used as both a trigger to activate the uncertain quantification procedure and the steepest increment direction to develop a fast searching phase. The stochastic approximation method is employed to minimize the computational burdens in computing the gradients. The numerical results on a case study are reported to validate the proposed methodology.

***Index Terms*—Inverse problem, robust optimization, stochastic approximation, tabu search method, uncertainty.**

I. Introduction

TRADITIONALLY, in the performance-based design optimizations of inverse problems, the goal is to find the global optimal solution according to the performance (objective) quality. However, imprecision and uncertainty are often inevitable and unavoidable in an engineering design problem. Hence, if the optimized solution is very sensitive to small variations of the optimized decision parameters, it is possible that slight perturbations in the optimized variables could result in either significant performance degradation or an infeasible solution due to the violation of the design constraint functions. The preferred practical design is thus not the global optimal solution in terms of only the performance (objective) function, but the one/ones that has/have high performances in both objective function and robustness in their design parameters against uncertainties. In this regard, it is equally important to explore robust optimal techniques in the studies of inverse problems under conditions of uncertainties in electromagnetic [1],[2].

Robustness means some degree of insensitivity to small perturbations in either the design or environmental variables. To quantify the uncertainty in robust design optimizations, numerous efforts have been reported [3]. Generally, there are two categories of uncertainty quantization, the probability-based and interval-based approaches [4]. The probability-based approach uses probabilistic information of the uncertainty, commonly the mean (expected fitness) and the standard deviation as the gauge to assess the robustness of a solution. However, the distribution probability of the uncertainties is unknown in the early stage of the design procedure. Also, the robustness of the final solution obtained using this approach cannot be guaranteed completely owing to the intrinsic properties of certain probability distributions [4]. The interval-based method simply uses the nominal value and the bounds of the uncertain parameters. Moreover, the interval uncertainty is very common in an engineering problem. In this regard, the interval-based approach is a more practical and convenient method in uncertainty quantization for practical inverse problems. Nevertheless, only lukewarm efforts are given to the development of robust design methodologies in computational electromagnetic. In this regard, a new methodology based on interval-based approach for uncertainty quantization is proposed and tested on a case study with promising results.

II. A New Methodology for Robust Optimizations

A constrained minimization problem with interval uncertainty can be formulated as

$$\begin{aligned} &\text{Min} \quad f(x,\delta) \\ &\text{S.T.} \;\; g_i(x,\delta) \le 0 \quad (i = 1,2,\ldots,k)\,, \\ &\qquad \delta_N - \delta_0 \le \delta \le \delta_N + \delta_0 \end{aligned} \tag{1}$$

where, x is the vector of design (decision) parameters (variables), δ is the vector of uncertainty variables, δ_N is the nominal value of δ, δ_0 is the half range of the interval uncertainty.

The robust counterparts of the objective function $f(x,\delta)$ and the constraint function $g_i(x,\delta)$ for the interval uncertainty using the worst case scenario are, respectively, defined as

$$f_w(x) = \underset{(\delta_N - \delta_0 \le \delta \le \delta_N + \delta_0)}{Max} [f(x,\delta)] \tag{2}$$

$$(g_i)_w(x) = \underset{(\delta_N - \delta_0 \le \delta \le \delta_N + \delta_0)}{Max} [g_i(x,\delta)] \tag{3}$$

In existing robust optimization methodologies [1],[2], the robust performances, such as those as defined in (2) and (3), are generally used in lieu of the original performance function to ensure the performance robustness of a final optimal solution. However, such formulations will result in a deficiency in the biasing force selection; and as a result, a different robust methodology is proposed and used in this paper.

A. Formulation of the Robust Optimization

To eliminate the shortcomings of both existing robust optimal methodologies and probability-based uncertainty quantization approaches, the robust performances are imposed as constraints in this paper. Moreover, to incorporate *a priori* knowledge of a domain expert, some acceptable tolerance for performance degradations is introduced. As a result, the proposed robust optimization formulation is formulated as

$$\begin{array}{l} \text{Min} \quad f(x, \delta_N) \\ \text{S.T} \;\; g_i(x, \delta_N) \le 0 \quad (i = 1, 2, \ldots, k) \end{array}, \tag{4}$$

together with

$$\begin{cases} (g_i)_w(x) - g_i(x, \delta_N) \le (\Delta g_i)_{tolerance} \\ f_w(x) - f(x, \delta_N) \le (\Delta f)_{tolerance} \end{cases} \; (\delta_N - \delta_0 \le \delta \le \delta_N + \delta_0) \tag{5}$$

where, $(\Delta g_i)_{tolerance}$ is the acceptable tolerance of constraint function $g_i(x, \delta)$, $(\Delta f)_{tolerance}$ is the acceptable tolerance of the objective function $f(x,\delta)$.

B. A Robust Oriented Tabu Search Algorithm

It should be pointed out that it is readily to use any evolutionary algorithm to find the robust optimal solution of (4) and (5). Nevertheless, the robust-oriented tabu search method [5], together with some specially designed mechanism for efficient robust optimizations, is used.

As explained in [5], the robust optimal solution of a constrained optimal design is either one of the local/global optima of the objective function or that distributed on the boundaries of the feasible parameter space. In this point of view, it is unnecessary to check (5) for every neighborhood solution. However, it is not easy to identify if a neighborhood solution is an optimal one during the optimization process. To address this problem, the (partial) derivative information is used as both a trigger to activate the validating procedures of (5) and the steepest increment direction to develop a fast searching phase for finding $(g_i)_w(x)$ and $f_w(x)$.

In the proposed mechanism, once the best solution, x^*, in the current neighborhood solutions, is identified, the (partial) derivative ($\nabla f(x^*)$) computation procedure will be activated for x^*, and a fast searching phase for finding $(g_i)_w(x)$ and $f_w(x)$ will then be implemented iff $\|\nabla f(x^*)\|_2$ is smaller than a predefined value. To overcome the disadvantage of computing only one partial derivative in one computation of the conventional finite-difference approach, the stochastic approximation method as introduced in [6] is used.

In the stochastic approximation method, a n-dimensional random vector (n is the dimension of the decision parameters), $[\Delta_1^{-1}\,\Delta_2^{-1}\cdots\Delta_n^{-1}]^T$, consisting of independent and symmetrically distributed entries with finite inverse moment expectation, is firstly generated using the symmetric Bernoulli (±1) distributions; and the gradient information for all n-dimensions is approximated at one computation using

$$\begin{aligned} \nabla f(x_k) &= \frac{\partial f(x)}{\partial x}\Big|_{x=x_k} \\ &= \frac{f_{robust}(x_k + c\Delta) - f_{robust}(x_k - c\Delta)}{2c}[\Delta_1^{-1}\,\Delta_2^{-1}\cdots\Delta_n^{-1}]^T \end{aligned} \tag{6}$$

Moreover, the proposed fast searching phase uses the gradient as the steepest increment direction for finding $(g_i)_w(x)$ and $f_w(x)$. Similarly, the gradients are computed use the same methodology of (6).

III. NUMERICAL APPLICATIONS

To validate the proposed robust optimal methodology, it is used to solve different case studies and compared with other existing robust optimization approaches. Due to space limitations, only the numerical results on the robust optimal counterpart of the Team Workshop problem 22 of the superconducting magnetic energy storage (SMES) configuration with three free parameters [7] are reported.

In the numerical study, the interval uncertainty is set to $\pm 1\%$ limit on the decision variables, and the acceptable tolerances for the stay field and the stored energy are set, respectively, as 35% and 0.15%. For performance comparisons, this case study is solved, respectively, by the proposed, the Combined Polynomial Chaos and PSO approach (CPC_PSO) for robust optimizations [8], and a general purpose tabu search algorithm. The numerical results will be reported in details in the full paper, and some observations are summarized as:

(1) The final solutions searched by the proposed methodology and the CPC_PSO are nearly the same. However, the computational time used by the former is only about 75% of that of the latter;

(2) The computational time used by the proposed methodology is about 1.2 times of that of the general purpose tabu search method although some time consuming procedures for robust performance qualifications are integrated in the proposed optimizer as an inner loop;

(3) For a $\pm 1\%$ limit of interval uncertainty on the optimized decision variables, the performance degradations of the global optimal solution obtained using the general purpose tabu search algorithm are 82% for the stay field and 2% for the stored energy; which are compared to 34.5% for the stay field and 0.14% for the stored energy of the robust optimal solution obtained using the proposed methodology.

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